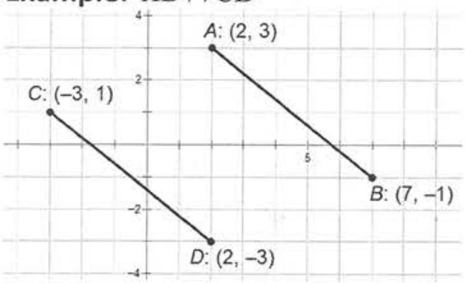
Geometry – Mr. Bo
Unit 6 – Day 5 Notes

Name: ______________________
Dates: ____________

Coordinate Geometry Proofs

Slope Formula $\frac{y_2 - y_1}{x_2 - x_1} = m$

Example: $\overline{AB} \,//\, \overline{CD}$

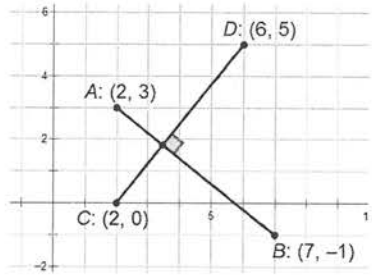


Slope AB = $\frac{3 - -1}{2 - 7} = \frac{4}{-5}$

Slope CD = $\frac{1 - -3}{-3 - 2} = \frac{4}{-5}$

Conclusion: // lines have = slopes.

Example: $\overline{AB} \perp \overline{CD}$



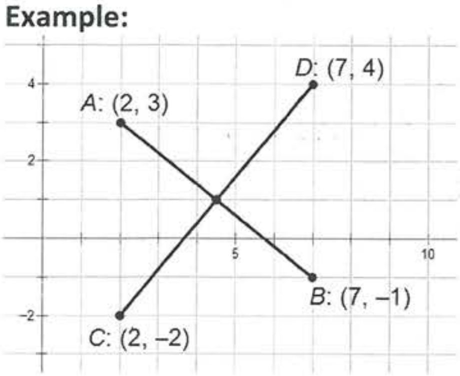
Slope AB = $\frac{3 - -1}{2 - 7} = \frac{4}{-5}$

Slope CD = $\frac{5 - 0}{6 - 2} = \frac{5}{4}$

Conclusion: ⊥ lines have neg. reciprocal slopes.

Midpoint Formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example:



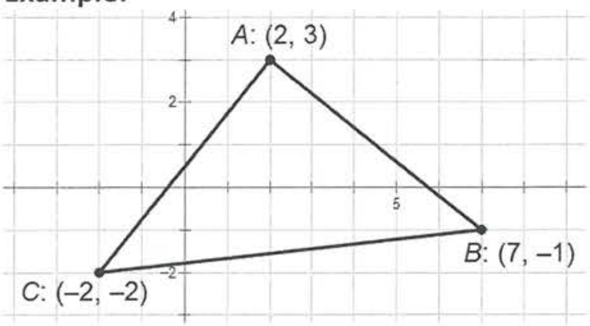
Midpoint AB = $\left(\frac{2 + 7}{2}, \frac{3 + -1}{2}\right) = \left(\frac{9}{2}, 1\right)$

Midpoint CD = $\left(\frac{7 + 2}{2}, \frac{4 + -2}{2}\right) = \left(\frac{9}{2}, 1\right)$

Conclusion: $\overline{AB}$ and $\overline{CD}$ bisect each other because they share the same midpt.

Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example:



AC = $\sqrt{(2 - -2)^2 + (3 - -2)^2} = \sqrt{16 + 25} = \sqrt{41}$

AB = $\sqrt{(2 - 7)^2 + (3 - -1)^2} = \sqrt{25 + 25} = \sqrt{41}$

Conclusion: $\overline{AC} \cong \overline{AB}$ because they have the same length.

Coordinate Geometry Proof: proof using Algebra formulas.

Steps:

1. Draw a picture.
2. use the formulas.

    Slope - for $\parallel$ or $\perp$

    Midpoint - for midpt or seg. bisector

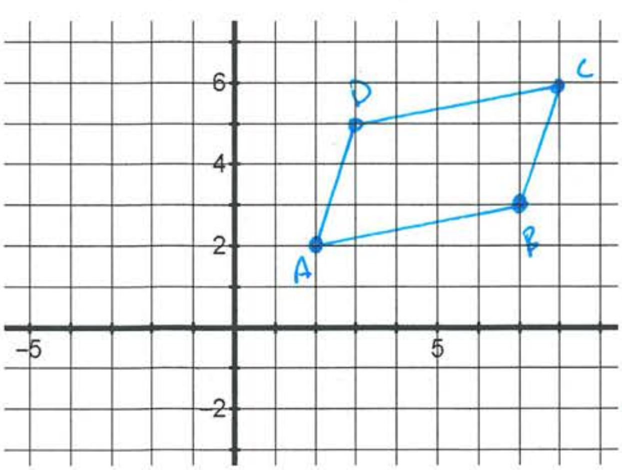
    Distance - for $\cong$ segments.

3. write a concluding sentence or two.

Example:

Given: Quadrilateral ABCD with A(2,2), B(7,3), C(8,6), D(3,5).

Prove: The opposite sides of ABCD are parallel



Slope AB = $\frac{3-2}{7-2} = \frac{1}{5}$

Slope DC = $\frac{6-5}{8-3} = \frac{1}{5}$

$\overline{AB} \parallel \overline{DC}$

Slope AD = $\frac{5-2}{3-2} = \frac{3}{1}$

Slope BC = $\frac{6-3}{8-7} = \frac{3}{1}$

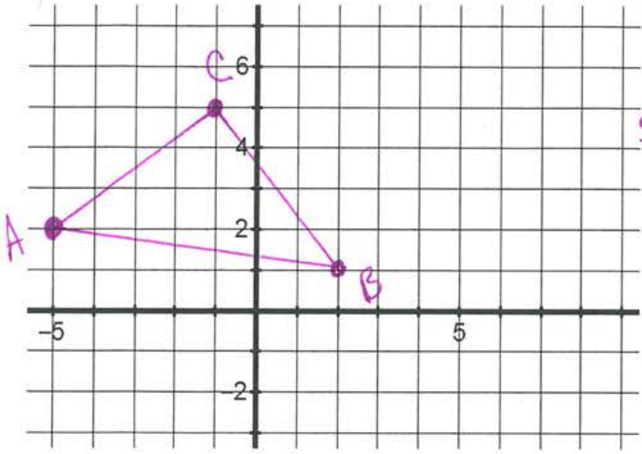
$\overline{AD} \parallel \overline{BC}$

$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ because they have = slopes.

Example:

Given: Triangle ABC with A(-5,2), B(2,1), C(~~-2,2~~)(-1,5).

Prove: Triangle ABC is an Isosceles Right triangle.



$$\text{Slope } AC = \frac{5-2}{-1--5} = \frac{3}{4}$$

$$\text{Slope } BC = \frac{5-1}{-1-2} = \frac{4}{-3}$$

$\overline{AC} \perp \overline{BC}$ because neg. reciprocal slopes.

$$AC = \sqrt{(-5--1)^2+(2-5)} = \sqrt{16+9} = \sqrt{25} = 5$$

$$BC = \sqrt{(2--1)^2+(1-5)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$\overline{AC} \cong \overline{BC}$ because equal lengths.

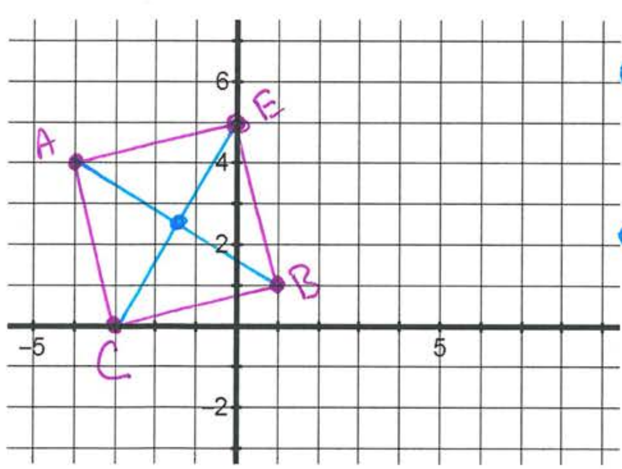
$\triangle ABC$ is Isosceles because it has a pair of $\cong$ sides.

$\triangle ABC$ is Right because it has a pair of $\perp$ sides.

Example:

Given: Quadrilateral ACBE with A(-4,4), C(-3,0), B(1,1), E(0,5).

Prove: The diagonals $\overline{AB}$ and $\overline{CE}$ bisect each other.



$$\text{midpt of } \overline{AB} = \left(\frac{-4+1}{2}, \frac{4+1}{2}\right) = \left(\frac{-3}{2}, \frac{5}{2}\right)$$

$$\text{midpt of } \overline{CE} = \left(\frac{-3+0}{2}, \frac{0+5}{2}\right) = \left(\frac{-3}{2}, \frac{5}{2}\right)$$

Since $\overline{AB}$ and $\overline{CE}$ share a midpt, they bisect each other.